"Different realizations of κ- momentum space and relative-locality effect"

S. Meljanac *,[1] A. Pachoł †,[2] A. Samsarov ‡,[1] and Kumar S. Gupta §[3]

[1] *Rudjer Bošković Institute, Bijenička c.54, HR-10002 Zagreb, Croatia*

[2] *Science Institute, University of Iceland, Dunhaga 3, 107 Reykjavik, Iceland*

[3] *Theory Division, Saha Institute of Nuclear Physics,*

1/AF Bidhannagar, Calcutta 700064, India

Abstract

We show that the different realizations for momentum sector of κ-Poincare Hopf algebra can be interpreted in the framework of a curved momentum space leading to the relativity of locality. The mass of a particle seems to be realization independent (up to linear order in deformation parameter l), therefore it indicates the existence of a universal Casimir element for a wide class of realizations. On the other hand, the time delay formula clearly shows a dependence on the choice of realization.

* meljanac@irb.hr
† pachol@hi.is
‡ asamsarov@irb.hr
§ kumars.gupta@saha.ac.in

I. INTRODUCTION

Recently postulated idea on relative locality [1–3] proposes to describe a "classical-non gravitational regime", where both $\hbar$ and G are negligible, but their ratio $M_P \sim \sqrt{\frac{\hbar}{G}}$ provides an energy scale given by the Planck mass. The emergence of such an energy scale provides the motivation to consider the momentum space as the fundamental entity and leads to the study of its geometry. Various features of this momentum space geometry can be described by a noncommutative algebra known as the κ-Minkowski algebra [4–7]. The κ-Poincare (quantum) symmetry group can be seen as one example of the origin for curved momentum space which provides the effect of the so-called "relative-locality". In addition, the κ-Minkowski algebra can be used to analyze the time-delay of signals coming from gamma ray bursts, which could be a signature of the Planck scale physics [8–13].

The noncommutative κ-Minkowski algebra and its symmetry quantum group is known in an infinite number of realizations in terms of commutative coordinates and derivatives [14–19]. In the context of the principle of relative locality, majority of the work done so far uses a particular realization of the κ-Poincare Hopf algebra, the so called Majid-Ruegg bicrossproduct realization. It is a natural question if other realizations can provide further insight into the consequences of relative locality. It might happen that different realizations point to a universality of certain physical results. On the other hand, if certain predictions depend on the choice of the realizations, that can be used to constrain the allowed class of realizations.

In this paper, we shall work within a particular class of realizations of the κ-Minkowski algebra that is much broader than just the single Majid-Ruegg bicrossproduct realization. We shall show that the linearized mass formula obtained from the geometry of the momentum space is independent of the realizations within the chosen class. On the other hand, the time delay in the observation of two particles emitted simultaneously depends explicitly on the choice of the realization. If such time delays can be experimentally measured, that would lead to phenomenological constraints on the allowed class of realizations of the κ-Minkowski algebra.

We start this Letter with the general κ-Poincare momenta realization which is used to obtain a general form of the metric on momentum space. Explicit calculation of the Christoffel symbols and geodesic equation in Sec.III are provided for the certain choice of realization, however still much broader than previously considered in the literature. Section IV starts with the deformed Poisson brackets which via

the Hamilton equations provide the solutions for worldlines, where the dependence on realisation is explicit as well as in the detection of arrival time for two particles. The concluding remarks close this Letter.

II. GENERALIZED METRIC

κ-Poincare inspired picture can be used as one of the illustrations of curved momentum-space geometry (as well as it provides an example of the energy-momentum sector of DSR theory). In [3] it was shown that by using the so-called Majid-Ruegg (bicrossproduct) realization for momenta one gets that the connection (parallel transport) is non-metric and torsion-full. However, one is not limited to this one basis of the κ-Poincare momenta sector and it is possible to consider the more general realization for the momenta, which can be written as [16],[17],[18]:

$$P_i = \frac{p_i}{\varphi(A)} Z^{-1}, \qquad P_0 = \frac{Z - Z^{-1}}{2l} - \frac{l}{2} Z^{-1} \frac{{p_i}^2}{\varphi^2(A)}, \qquad P_4 = \frac{Z + Z^{-1}}{2l} - \frac{l}{2} Z^{-1} \frac{{p_i}^2}{\varphi^2(A)} \tag{1}$$

for any ψ, φ. With the lorentzian metric $\eta_{\mu\nu} = (+,-,-,-)$ and the following notation: $A = ia \cdot \partial = -a \cdot p$ where we choose $a = (l, 0, 0, 0)$ and the quantities like $p_i^2 = p_i p_i$ $(i = 1, 2, 3)$, where summation over space indices is understood. Also in the above realizations we used $Z = e^{\Psi(A)}$ with $\Psi(A) = \int_0^A \frac{dt}{\psi(t)}$[1], where Z is the so-called shift operator which satisfies $\left[Z, p_\mu\right] = 0$.

Such coordinates $P_I = \left(P_\mu, P_4\right)$ (1) satisfy the (hyperboloid) condition [20]:

$$P_0^2 - P_1^2 - P_2^2 - P_3^2 - P_4^2 = -\frac{1}{l^2} \tag{2}$$

and provide the four dimensional de Sitter space which can be parametrized by p_μ.

From this point of view the space of momenta is not a flat space, as in Special Relativity, but it is curved, maximally symmetric space of constant curvature (this fact was already used within DSR framework , see e.g. [21]).

One can show that the general realization (1) for the κ -Poincare momenta describe a momentum

[1] The functions ψ, ϕ are related to different realizations of κ-Minkowski spacetime and will be discussed in Section IV.

space with the ’generalized de Sitter metric’ which leads to the ’relative-locality’ effect as well.

$$ds^2 = \left[-\frac{1}{l^2}\left(Z^{-1}\right)' Z' + \left(\left(Z^{-1}\right)'\left(\frac{1}{Z\varphi^2}\right)' - \left(\frac{1}{Z\varphi}\right)'^2\right)p_i^2\right]dp_0^2$$
$$-\left(\frac{1}{Z\varphi}\right)^2 dp_i^2 + 2\left(\left(Z^{-1}\right)' Z^{-1}\varphi^{-2} - \left(\frac{1}{Z\varphi}\right)'\frac{1}{Z\varphi}\right)p_i dp_0 dp_i$$

where $(\cdot)' = \frac{d}{dp_0}$. In fact the line element ds^2 above is a local expression for an induced metric on the hyperboloid (2) written in local coordinate system provided by the formula (1).

However to obtain the relative-locality effect (in the more general ’framework’ than in [3]), it is enough to consider the simpler cases, with the choice $\psi = 1$ for which the shift operator is $Z = e^{-lp_0} = e^A$, hence the realization of momenta reduces to:

$$P_0(p_0, p_i) = \frac{1}{l}\sinh(lp_0) + \frac{lp_i^2}{2\varphi^2}e^{lp_0}; \tag{3}$$

$$P_i(p_0, p_i) = \frac{p_i}{\varphi}e^{lp_0}; \tag{4}$$

$$P_4(p_0, p_i) = \frac{1}{l}\cosh(lp_0) - \frac{lp_i^2}{2\varphi^2}e^{lp_0}. \tag{5}$$

For this choice $\varphi_\lambda = Z^{-\lambda} = e^{-\lambda A} = e^{\lambda lp_0}$ (λ is real). Within this realization one gets the line element which depends on the parameter λ and has the form:

$$ds_\lambda^2 = \left[1 - l^2\lambda^2 p_i^2 e^{2(1-\lambda)lp_0}\right]dp_0^2 - e^{2(1-\lambda)lp_0}dp_i^2 + 2l\lambda e^{2(1-\lambda)lp_0}p_i dp_0 dp_i$$

Therefore the metric is as follows:

$$g_{\mu\nu} = \begin{pmatrix} 1 - l^2\lambda^2 p_i^2 e^{2(1-\lambda)lp_0} & l\lambda e^{2(1-\lambda)lp_0}p_1 & l\lambda e^{2(1-\lambda)lp_0}p_2 & l\lambda e^{2(1-\lambda)lp_0}p_3 \\ l\lambda e^{2(1-\lambda)lp_0}p_1 & -e^{2(1-\lambda)lp_0} & 0 & 0 \\ l\lambda e^{2(1-\lambda)lp_0}p_2 & 0 & -e^{2(1-\lambda)lp_0} & 0 \\ l\lambda e^{2(1-\lambda)lp_0}p_3 & 0 & 0 & -e^{2(1-\lambda)lp_0} \end{pmatrix} \tag{6}$$

One can easily notice that for the choice of $\lambda = 0$ we recover the Majid-Ruegg case[2]: $ds^2 = dp_0^2 - e^{2lp_0}dp_i^2$ with the so-called ’Majid-Ruegg metric’ $g_{\mu\nu} = diag\left(1, -e^{2lp_0}, -e^{2lp_0}, -e^{2lp_0}\right)$ [3].

[2] The convention in this letter differs from the one introduced in [3] by $l \to -l$.

III. CHRISTOFFEL SYMBOLS

From any metric one can calculate the Christoffel symbols from the general formula:

$$\Gamma^{\mu\nu}_{\rho} = \frac{1}{2} g_{\sigma\rho} \left(g^{\sigma\mu,\nu} + g^{\nu\sigma,\mu} - g^{\mu\nu,\sigma} \right) \tag{7}$$

Limiting ourselves to the case of $\psi = 1$, $\varphi = Z^{-\lambda} = e^{-\lambda A}$, the non-zero components of the metric (6) are:

$$g_{00} = 1 - l^2\lambda^2 p_i^2 e^{2(1-\lambda)lp_0}; \qquad g_{ki} = -e^{2(1-\lambda)lp_0}\delta_{ki}; \qquad g_{0i} = g_{i0} = l\lambda e^{2(1-\lambda)lp_0} p_i.$$

The inverse metric is:

$$g^{\rho\sigma} = \begin{pmatrix} 1 & l\lambda p_1 & l\lambda p_2 & l\lambda p_3 \\ l\lambda p_1 & l^2\lambda^2 p_1^2 - e^{-2(1-\lambda)lp_0} & l^2\lambda^2 p_1 p_2 & l^2\lambda^2 p_1 p_3 \\ l\lambda p_2 & l^2\lambda^2 p_1 p_2 & l^2\lambda^2 p_2^2 - e^{-2(1-\lambda)lp_0} & l^2\lambda^2 p_2 p_3 \\ l\lambda p_3 & l^2\lambda^2 p_1 p_3 & l^2\lambda^2 p_2 p_3 & l^2\lambda^2 p_3^2 - e^{-2(1-\lambda)lp_0} \end{pmatrix}$$

For this choice of realization in the metric we obtain the following set of Christoffel symbols:

$$\Gamma^{0j}_{i} = -\left(1-\lambda\right) l\delta^j_i = \Gamma^{j0}_{i}; \qquad \Gamma^{ij}_{0} = l\left(\lambda - (1-\lambda)\left(e^{-2l(1-\lambda)p_0} - l^2\lambda^2 p_i^2\right)\right)\delta^{ij}; \tag{8}$$

$$\Gamma^{i0}_{0} = l^2(1-\lambda)\lambda p^i = \Gamma^{0i}_{0}; \qquad \Gamma^{ij}_{k} = -l^2(1-\lambda)\lambda p_k \delta^{ij}; \tag{9}$$

$$\Gamma^{00}_{0} = 0; \qquad \Gamma^{00}_{k} = 0. \tag{10}$$

It can be seen that, within the first order in deformation, the components Γ^{0j}_0 and Γ^{ij}_k vanish

$$\Gamma^{0j}_{0} = \mathcal{O}(l^2); \qquad \Gamma^{ij}_{k} = \mathcal{O}(l^2). \tag{11}$$

For the sake of comparison with the results in Ref.[3], we give the explicit expressions of the above quantities for the special case of $\lambda = 0$:

$$\Gamma^{0j}_{i} = \Gamma^{j0}_{i} = -l\delta_{ij}; \qquad \Gamma^{ij}_{0} = -le^{-2lp_0}\delta^{ij}; \tag{12}$$

$$\Gamma^{0j}_{0} = 0; \qquad \Gamma^{ij}_{k} = 0. \tag{13}$$

A. Geodesic equation

In this chapter and later on we will consider only the first order in the deformation parameter l. The geodesic equation reads as:

$$\ddot{p}_{\rho} + \Gamma_{\rho}^{\mu\nu} \dot{p}_{\mu} \dot{p}_{\nu} = 0 \tag{14}$$

where $\dot{} = \frac{d}{ds}$ and s denotes a geodesic parametrization.

For the solution of the geodesic equation up to the first order in the deformation parameter 'l' we can use the following ansatz [3]

$$p_{\rho}(s) = P_{\rho} s + \frac{1}{2} \Gamma_{\rho}^{\mu\nu} P_{\mu} P_{\nu} \left(s - s^2\right)$$

$$\dot{p}_{\rho}(s) = P_{\rho} + \frac{1}{2} \Gamma_{\rho}^{\mu\nu} P_{\mu} P_{\nu} (1 - 2s)$$

with the initial conditions: $p_{\mu}(0) = 0$; $\dot{p}_{\mu}(1) = P_{\mu}$.

Also the inverse metric in the linear order in l has the easier form

$$g^{\rho\sigma} = \begin{pmatrix} 1 & l\lambda p_1 & l\lambda p_2 & l\lambda p_3 \\ l\lambda p_1 & -1 + 2(1-\lambda) l p_0 + O(l^2) & 0 & 0 \\ l\lambda p_2 & 0 & -1 + 2(1-\lambda) l p_0 + O(l^2) & 0 \\ l\lambda p_3 & 0 & 0 & -1 + 2(1-\lambda) l p_0 + O(l^2) \end{pmatrix}$$

There are only two non-zero Christoffel symbols in this case:

$$\Gamma_i^{0j} = -(1-\lambda) l \delta_i^j; \qquad \Gamma_0^{ij} = l(2\lambda - 1)\delta^{ij} \tag{15}$$

Therefore our solutions read as follows:

$p_0(s) = P_0 s + \frac{l}{2}(2\lambda - 1) P_i^2 \left(s - s^2\right)$ with $\dot{p}_0(s) = P_0 + \frac{l}{2}(2\lambda - 1) P_i^2 (1 - 2s)$

and $p_i(s) = P_i s - (1-\lambda) l \delta_i^j P_0 P_j \left(s - s^2\right)$ with $\dot{p}_i(s) = P_i - (1-\lambda) l \delta_i^j P_0 P_j (1 - 2s)$.

With this, it is straightforward to calculate the quadratic expression $g^{\mu\nu} \dot{p}_{\mu}(s) \dot{p}_{\nu}(s) = P_0^2 - P_i^2 + l P_0 P_i^2 + O(l^2)$, giving rise to the length of the momentum space worldline. Indeed, the length of the worldline, $D\left(0, P_{\mu}\right) = \int_0^1 ds \sqrt{g^{\mu\nu} \dot{p}_{\mu}(s) \dot{p}_{\nu}(s)}$, in momentum space between the two boundary points, specified by the two values of the parameter s, namely 0 and 1 respectively, can be calculated within the first order in deformation l as

$$D\left(0, P_{\mu}\right) = \int_0^1 ds \sqrt{P_0^2 - P_i^2 + l P_0 P_i^2} = \sqrt{P_0^2 - P_i^2 + l P_0 P_i^2}. \tag{16}$$

Postulating that the geodesic distance from the origin to a generic point in momentum space is the mass of a particle [1], we get the relation:

$$m^2 = P_0^2 - P_i^2 + lP_0P_i^2 + \mathcal{O}(l^2) \tag{17}$$

The obtained result is the same as in [3], therefore it is realization independent, i.e. there is no explicit dependence on λ. Result of this calculations show that the above postulate is physically reasonable, since the Casimir should not depend on the choice of the ordering nor realization (relation between ordering and realizations is discussed in [16]). Nevertheless, it seems that the physical phenomena, as the time delay, will depend on realization for the noncommutative coordinates, at least within the class of realizations considered in this paper, parametrized by the parameter λ. And this point will be shown in the next chapter.

IV. HAMILTONIAN DESCRIPTION AND TIME DELAY

Above introduced momenta realisation corresponds to a certain realisation of noncommutative (κ-Minkowski) spacetime coordinates:

$$\hat{x}_0 = x_0\psi(A) - lx_kp_k\gamma(A), \quad \hat{x}_i = x_i\varphi(A) \tag{18}$$

for an arbitrary choice of ψ, φ, where φ is the same function appearing in the momentum realization (1). This functions satisfy: $\gamma = \frac{\varphi'}{\varphi}\psi + 1$ with the initial conditions: $\psi(0) = \varphi(0) = 1, \varphi'(0)$-finite and $A = ia \cdot \partial = -a \cdot p$. (with $a = (l, 0, 0, 0)$ as before) with $\varphi' = \frac{\partial\varphi}{\partial A}$.

A special case of the above, when one chooses: $\varphi = Z^{-\lambda}$; $\psi = 1$; $\gamma = (1 - \lambda)$ and

$$\hat{x}_0 = x_0 - l(1-\lambda)x_kp_k, \quad \hat{x}_i = x_iZ^{-\lambda} \tag{19}$$

will be used in the calculations below.

Such realization (18,19) satisfy the following (κ-Minkowski) commutation relations:

$$[\hat{x}_0, \hat{x}_i] = il\hat{x}_i; \qquad [\hat{x}_i, \hat{x}_k] = 0 \tag{20}$$

κ -deformed phase space with deformed Poisson brackets can be obtained by the so-called "dequantization" procedure: $\{\ ,\ \} = \frac{1}{i}[\ ,\ ]$. In this way we obtain:

$$\{x_0, x_i\} = lx_i, \qquad \{x_i, x_j\} = 0 \tag{21}$$

together with

$$\{p_0, x_0\} = 1; \qquad \{p_0, x_i\} = 0; \tag{22}$$

$$\{p_i, x_0\} = l\,(1-\lambda)\,p_i; \qquad \{p_i, x_j\} = -e^{\lambda l p_0}\delta_{ij}. \tag{23}$$

It is easy to see that the realizations (19) in conjunction with the ordinary Heisenberg algebra $\left[p_\mu, x_\nu\right] = i\eta_{\mu\nu}$ lead to a phase space commutation relations, which through the above described dequantisation procedure come up with the momentum space Poisson brackets (22) and (23).

Previously obtained linearized relation $m^2 = p_0^2 - p_i^2 + l p_0 p_i^2$ can be used to postulate the form of the Hamiltonian [23, 24] as:

$$\mathcal{H} = \mathcal{N}\left(p_0^2 - p_i^2 + l p_0 p_i^2 - m^2\right)$$

where $\mathcal{N}$ is the constant multiplier. Even though the on-shell relation (17) does not depend on the realisation, the parameter λ will enter the particle's velocity and worldline through the Poisson brackets (23). This is made obvious by writing down the Hamilton equations for the particle coordinates, which give rise to [3]:

$$\dot{x}_0 = -\mathcal{N}\left(2p_0 + l p_1^2 + (2l p_0 p_1 - 2p_1)\,l\,(1-\lambda)\,p_1\right); \tag{24}$$

$$\dot{x}_1 = -2\mathcal{N}\,(l p_0 p_1 - p_1)\,e^{\lambda l p_0}, \tag{25}$$

with the corresponding equations for the particle momenta being trivial. This leads to the velocity of a particle (in general):

$$v = \frac{2\,(l p_0 p_1 - p_1)\,e^{\lambda l p_0}}{2p_0 + l p_1^2 + (2l p_0 p_1 - 2p_1)\,l\,(1-\lambda)\,p_1}$$

and in the leading order in l:

$$v = -\frac{p_1}{\sqrt{m^2 + p_1^2}} - (\lambda - 1)\,l p_1 \frac{m^2}{m^2 + p_1^2} + O(l^2).$$

Therefore the worldline of the particle appears to be given by

$$x^1 = \bar{x}^1 + v\left(x^0 - \bar{x}^0\right) = \bar{x}^1 - \left(\frac{p_1}{\sqrt{m^2 + p_1^2}} + (\lambda - 1)\,l p_1 \frac{m^2}{m^2 + p_1^2}\right)\left(x^0 - \bar{x}^0\right) \tag{26}$$

[3] for simplicity we consider 1+1 dim case

where $\bar{x}^0, \bar{x}^1$ are the initial time and position, respectively.

One can notice that the worldline for the massless particle is momentum and realisation independent:

$$x^1 = \bar{x}^1 - \frac{p_1}{|p_1|}\left(x^0 - \bar{x}^0\right).$$

However this fact does not imply that simultaneous emission of such particles with different momenta will be detected simultaneously [22]. This appears to be one of the properties of relative locality idea. Following the analogous analysis to the one performed in [3], we obtain the correction to the difference of Bob's detection times for the two particles sent by Alice:

$$\Delta t = lb\,(1-\lambda)\,\Delta p_1$$

where b is the distance between Alice and Bob and Δp_1 is the momentum difference between two photons emitted from the position of Alice (cf. [13]).

One can notice that for λ=0 (right-ordering) we recover the result from [3], while for λ=1, the case which corresponds to the left-ordering, there is no Planck scale effect at all.

V. CONCLUSION

In this Letter we have considered a large class of realizations of the momentum sector of κ-Poincare algebra and have studied the effect of the variation of realizations on the expressions for the mass as well as the time delay formulae as obtained within the DSR framework. The mass formula obtained in [3] using the Majid-Ruegg bicrossproduct realization agrees with that obtained in this Letter. This indicates the existence of a universality in the mass formula for a wide class of realizations. On the other hand, the time delay formula clearly shows a dependence on the choice of realization. This is interesting from a phenomenological point of view, since observations of time delays of signals coming from a GRB can be used to put constraint on the allowed class of realizations.

A particular choice of the ordering prescription may also appear to be important in other physical contexts, such as that of quantum statistics. This was demonstrated to be the case by mutual comparison of the oscillator algebras obtained in a number of different works [25],[26],[27],[28],[29],[30],[31]. However, from this perspective, it is quite interesting to note that for a class of orderings/realizations of the κ-Minkowski space considered in this paper, there exists a universal R-matrix, the same for all

realizations within this class, leading to the same algebra of creation and annihilation operators appearing in the mode expansion of the field operator and consequently leading to the same particle statistics. What would be even more intriguing is to have this R-matrix fully expressed in terms of the Poincare generators, which would provide a unique covariant definition of the particle exchange, as well as the covariant notion of identical particles in the κ-deformed field theories. Some progress in this direction has been done in Refs.[32],[33]. Another issue is the choice of the metric on the deformed momentum space. Within the introduced framework, it would be interesting to investigate if, e.g. metric in momentum space introduced via the commutation relations of the deformed Lorentz generators [34] would also lead to the similar relative-locality effects or what would be the mass relation calculated via geodesic distance.

Aknowledgements

We would like to thank A. Borowiec for helpful remarks and discussion. S.M. and A.S work was supported by the Ministry of Science and Technology of the Republic of Croatia under contract No. 098-0000000-2865. A.P. acknowledges the financial support of Polish NCN grant No.2011/01/B/ST2/03354.
Note added: We were informed by G. Amelino-Camelia that he and his group is analyzing similar issues.

---

[1] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Phys. Rev. D84 (2011) 084010 [arXiv:1101.0931].

[2] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman L. Smolin, General Relativity and Gravitation 43 (2011) 2547 [arXiv:1106.0313].

[3] G. Amelino-Camelia, M. Arzano, J. Kowalski-Glikman, G. Rosati, G. Trevisan, Class. Quantum Grav. 29 (2012) 075007 [arXiv:1107.1724].

[4] J. Lukierski, A. Nowicki, H. Ruegg and V. N. Tolstoy, Phys. Lett. **B264**, 331 (1991).

[5] J. Lukierski, A. Nowicki and H. Ruegg, Phys. Lett. **B293**, 344 (1992).

[6] J. Lukierski and H. Ruegg, Phys. Lett. **B329**,189 (1994) [arXiv:hep-th/9310117].

[7] J. Lukierski, H. Ruegg and W. J. Zakrzewski, Ann. Phys. **243**, 90 (1995).

[8] A. Abdo et al, Nature **323**, 1688 (2009).

[9] A. Abdo et al, Nature **462**, 331 (2009).

[10] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. **A 12**, 607 (1997).

[11] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos and D. V. Nanopoulos and S. Sarkar, Nature **393**, 763 (1998).

[12] G. Amelino-Camelia and L. Smolin, Phys. Rev. **D 80**, 084017 (2009).

[13] A. Borowiec, Kumar S. Gupta, S. Meljanac and A. Pachol, Europhys. Lett. **92**, (2010) 20006.

[14] S. Meljanac and M. Stojic, Eur. Phys. J. C **47**, 531 (2006) [arXiv:hep-th/0605133].

[15] S. Meljanac, S. Kresic-Juric and M. Stojic, Covariant realizations ofκ-deformed space, Eur. Phys. J. **C51**, 229 (2007) [arXiv:hep-th/0702215].

[16] S. Meljanac, A. Samsarov, M. Stojic, K. S. Gupta Eur.Phys.J.C53:295-309 (2008) [arXiv:0705.2471].

[17] S. Meljanac, S. Kresic-Juric Int. J. Mod. Phys. A 26 (20) (2011) 3385 [1004.4647];

[18] A. Borowiec, A. Pachoł, SIGMA 6:086 (2010); [arXiv:1005.4429].

[19] A. Borowiec, A. Pachoł, Phys.Rev.D79:045012 (2009); [arXiv:0812.0576]

[20] J. Kowalski-Glikman, S. Nowak; [arXiv:hep-th/0411154]

[21] J. Kowalski-Glikman, Phys.Lett. B547 (2002) 291-296 [arXiv:hep-th/0207279].

[22] G. Amelino-Camelia, N. Loret and G. Rosati , Phys. Lett. B700 (2011) 150 [arXiv:1102.4637].

[23] M. Daszkiewicz, K. Imilkowska, J. Kowalski-Glikman, Phys.Lett. A323 (2004) 345-350, [arXiv:hep-th/0304027].

[24] S. Mignemi, Phys.Rev. D68 065029 (2003) [arXiv:gr-qc/0304029].

[25] M. Daszkiewicz, J. Lukierski and M. Woronowicz, Mod. Phys. Lett. A **23** (2008) 653 [hep-th/0703200].

[26] M. Arzano and A. Marciano, Phys. Rev. D **76** (2007) 125005 [arXiv:0707.1329].

[27] M. Daszkiewicz, J. Lukierski and M. Woronowicz, Phys. Rev. D **77** (2008) 105007 [arXiv:0708.1561].

[28] C. A. S. Young and R. Zegers, Nucl. Phys. B **797** (2008) 537 [arXiv:0711.2206].

[29] T. R. Govindarajan, Kumar S. Gupta, E. Harikumar, S. Meljanac, D. Meljanac, Phys.Rev.D77:105010,2008 [arXiv:0802.1576].

[30] C. A. S. Young and R. Zegers, Nucl. Phys. B **804** (2008) 342 [arXiv:0803.2659].

[31] T. R. Govindarajan, Kumar S. Gupta, E. Harikumar, S. Meljanac, D. Meljanac, Phys.Rev.D80:025014,2009 [arXiv:0903.2355].

[32] C. A. S. Young and R. Zegers, Nucl. Phys. B **809** (2009) 439 [arXiv:0807.2745].

[33] S. Meljanac, A. Samsarov and R. Strajn, JHEP **1208** (2012) 127 [arXiv:1204.4324].

[34] D. Kovacevic, S. Meljanac, A. Pachol, R. Strajn, Phys. Lett. B 711, 122 (2012); [arXiv:1202.3305].